Chapter 38 Diffraction

Answers to Understanding the Concepts Questions

1. The locations of the diffraction minima are given by $\sin\theta = m\lambda/a$. As a is reduced θ increases, so the diffraction pattern spreads out. The width of the central maximum, for example, expands. The pattern also gets dimmer overall, as less light is getting through the narrower slit.

2. For a fixed wavelength, the angle of diffraction decreases as the obstacle size increases. Thus a thinner support pole will give rise to an increased diffraction pattern. In the limit of a support pole that is very small compared to the wavelength, the waves will pass around the pole as if it were not there.

3. The Moon does not have an atmosphere that can distort the light from celestial objects.

4. The bright spot that we refer to as the Poisson spot is due to the constructive interference between the coherent light coming from the rim of the obstacle. In contrast to a flat disk, a coherent light source that is close to a bowling ball will have rays of grazing incidence on the side of the ball that is invisible from the screen. The waves would have to diffract a good way around the ball, and the result is that the intensity is highly reduced. There is little hope of seeing the Poisson spot unless the source is very far away, effectively at infinity. By the same token, light that originates on an equatorial circle of a sphere cannot be focused onto a near screen unless a lens that focuses parallel rays onto the screen is used. Without a lens, the screen too must be far away.

5. A shorter wavelength would be the choice, as it allows the laser to inscribe with closer spacing and therefore "squeezing" for information onto the same disc area.

6. As a general principle, we can never resolve any details whose dimensions are smaller than the wavelength of the light being used for the observation. Therefore, the shorter the wavelength of the light, the higher its resolution power. Blue light is better than red light for this purpose.

7. No. If the first wave is in the x-direction while the second one is on the y-direction, then the combined wave satisfies $E^2 = E_1^2 + E_2^2$. Since the time average of both E_1^2 and E_2^2 are uniform, so is E^2. The intensity of the combined wave, which is proportional to the time average of E^2, is therefore uniform, with no variation over space. There is no interference. This is to be differentiated with the case when the two waves are polarized in the same direction, in which case $E^2 = (E_1 + E_2)^2 = E_1^2 + E_2^2 + 2E_1E_2$. It is the extra cross term, $2E_1E_2$, that gives rise to the phenomenon of interference when the two waves are coherent.

8. To observe a stable interference and/or diffraction pattern, the participating light rays must be coherent. That usually means that the path difference between them should be of the order of the wavelength, (around a micron). Laser is an exceptionally coherent source so that condition can be relaxed. You can, for example, put a rubber band of width around a millimeter in the path of a He-Ne laser, and obtain a diffraction pattern on a screen. A hair would do better. For a ruler, let's assume that the interference and diffraction patterns are caused as a result of the laser beam striking various etch marks on the ruler. If you are looking for the interference from the various millimeter marks, the beam has to be at a glancing angle, so that the interference fringes are not terribly close to each other, which would make them difficult to observe. This also makes it possible for a thin laser beam to bounce off a

number of such marks. The diffraction, on the other hand, is caused by the phase difference of light rays that are reflected off the various parts of the same etch mark. Since the mark is much narrower than a millimeter, glancing incidence is not quite necessary.

9. The relationship between the angle subtended by the central bright spot that limits the resolution and the wavelength of the light is $\theta_{\min} = \lambda / D$, where D is the size of the aperture. If an instrument has fixed aperture, the angular size of the bright spot is therefore larger if the wavelength is larger. Red light has a longer wavelength than blue light, so that the resolution is worse if red light is used.

10. No. According to the textbook, the holographic film is essentially a diffraction grating. When light is diffracted through this grating, a three-dimensional holographic image is produced. Taking only a part of the film is equivalent to using a shorter grating with less number of lines. The resolution power, which is proportional to the total number of lines in the grating, is reduced.

11. The centerpiece blocks all the waves except the ones from the two narrow slits. If these two waves are in phase then we have a maximum. If the centerpiece is removed then we need to sum up the contribution of the waves not only from these two slits, but everywhere in between as well.

12. Figure 38-8 illustrates the differences. The locations of the principal maxima are exactly the same in the two cases. The intensities of these maxima are much larger for large N than for the double slit, while their widths are much smaller so their positions are well-defined. It is these differences that make the N-slit pattern so much more useful for the measurement of wavelength.

13. The spectrum of light emitted from a blackbody is continuous, with a peak centered at a certain wavelength that is determined by the surface temperature of the blackbody.

14. There is a component of diffraction in what happens to ocean waves that pass around you, but most of it is just hydrodynamic flow. To see pure diffraction effects due to an obstacle, you need waves that involve a disturbance of a stationary medium. If you were to stand in a still pond into which somebody dropped a stone, your body would diffract the resulting surface waves as they pass around you.

15. The X-ray wave interacts with each atom to produce a diffracted wave. When all the diffracted waves are combined, there are certain directions in which the waves enhance each other, and these directions can be determined through the arrangement of atomic planes.

16. A rough estimate is best made with the formulas that describe the diffraction due to a single slit. For visible light, the wavelength λ is around 4×10^{-7} m. For a diffraction angle θ such that $\sin\theta$ is of the order of one, the intensity of the diffracted light is roughly $(\lambda / d)^2$, where I_0 is the intensity of the undiffracted light and d is the slit width. For any reasonable slit width this is a very tiny factor.

17. As described in the question statement, these "3-D" images work by sending slightly different 2-D pictures to the left and right eyes, which is exactly how we normally perceive images to be 3-D — In fact if you close one eye you will have a hard time judging distances since you lose the ability to compare between the pictures received by both eyes. This is very different from a holographic image, however; since if you were to tilt your head and observe the image from different angles, only a holographic image is fully 3-D, in that it reveals different details of the original 3-D object at different angles.

18. The intensity in the middle of the diffraction pattern on the screen depends on the amplitude of the electric field there, which is the sum of the contribution from all parts of the circular opening. At some distance between the two screens the sum happens to be zero, and we get a dark spot in the middle of the pattern. As the screen distance is increases the value of the sum also changes, alternating between maxima and minima. Correspondingly, the center of the diffraction pattern brightens up and darkens.

Eventually, as the screen distance becomes sufficiently large, the light rays from various parts of the circular opening are essentially parallel, and their phase differences diminish. So the waves are essentially in phase as they arrive at the center of the pattern, which is always bright now.

Solutions to Problems

Note: For problems involving the resolution of the diffraction pattern of a circular aperture, we will use the approximation $\theta_{\min} \simeq \lambda/D$.

1. Because the lines to the maxima are at small angles from the line to the central peak, we have
$\sin\theta \simeq \tan\theta = y/L$.
We find the wavelength from the condition for the maxima:
$d\sin\theta \simeq d\tan\theta = dy/L = m\lambda, \quad m = 0, \pm 1, \pm 2, \dots;$
$[1/(400 \text{ lines/cm})](10^{-2} \text{ m/cm})(10.34 \times 10^{-2} \text{ m})/(1.44 \text{ m}) = 4\lambda$, which gives $\lambda = 4.5 \times 10^{-7} \text{ m} = \boxed{450 \text{ nm}}$.

2. The maxima are given by
$d\sin\theta \simeq d\tan\theta = dy/L = m\lambda$, or $y = (mL/d)\lambda, \quad m = 0, \pm 1, \pm 2, \dots;$
For two wavelengths that differ by $\Delta\lambda$, we have
$\Delta y = (mL/d)\,\Delta\lambda;$
$\Delta y = [2(1.5 \text{ m})(1200 \text{ lines/cm})(10^2 \text{ cm/m})][(635 - 620) \times 10^{-9} \text{ m}]$, which gives
$\Delta y = 5.4 \times 10^{-3} \text{ m} = \boxed{5.4 \text{ mm}}$.

3. The maxima are given by
$d\sin\theta \simeq d\tan\theta = dy/L = m\lambda, \quad m = 0, \pm 1, \pm 2, \dots;$
$d\tfrac{1}{2}(14.3 \text{ cm})/(265 \text{ cm}) = 10(680 \times 10^{-9} \text{ m})$, which gives $d = 2.52 \times 10^{-4}$ m.
The spacing is
$1/d = 1/(2.52 \times 10^{-4} \text{ m}) = 3.97 \times 10^3 \text{ lines/m} = \boxed{39.7 \text{ lines/cm}}$.

4. (a) The resolution power is
$R = mN = \boxed{2N}$, where N is the total number of lines in the grating.
(b) Since $d\sin\theta = m\lambda$, for $m = 2$
$\sin\theta = m\lambda/d \approx 2(589.3 \text{ nm})/(1 \text{ cm}/4500) = 0.5304$, or $\theta \approx 32°$.
To find the angular separation, note that
$d\,\Delta\sin\theta \approx d\cos\theta\ \Delta\theta = m\,\Delta\lambda$, so
$\Delta\theta \approx m\,\Delta\lambda/\,d\cos\theta = 2(589.6 \text{ nm} - 589.0 \text{ nm})/[(1\text{cm}/4500)\ \cos\ 32°] = \boxed{6.4 \times 10^{-4} \text{ rad}}$.

5. The resolving power needed is
$R = \lambda/\Delta\lambda = mN,$
so the number of lines per centimeter for the grating of total length L is
$N/L = \lambda/mL\,\Delta\lambda = (422.8 \text{ nm})/[3(0.800 \text{ cm})(422.787 \text{ nm} - 422.729 \text{ nm})] = \boxed{3.0 \times 10^3 \text{ lines/cm}}$.

6. (a) We find the angle for the second order from
$d\sin\theta = m\lambda;$
$[(3 \times 10^{-2} \text{ m})/(2.0 \times 10^4)]\sin\theta = 2(530 \times 10^{-9} \text{ m})$, which gives $\sin\theta = 0.707$, $\theta = 45°$.
We find the dispersion from
$\Delta\theta/\Delta\lambda = m/(d\cos\theta)$
$= 2/[(3.0 \times 10^{-2} \text{ m})/(2.0 \times 10^4)]\cos 45° = 1.9 \times 10^6 \text{ rad/m} = \boxed{1.9 \times 10^{-3} \text{ rad/nm}}$.
(b) We find the smallest-wavelength interval from the resolution:
$R = \lambda/\Delta\lambda = mN;$
$(530 \text{ nm})/\Delta\lambda = 2(2.0 \times 10^4)$, which gives $\Delta\lambda = \boxed{0.013 \text{ nm}}$.

7. We find the resolving power from

$R = mN$;

$R_1 = 1(5000 \text{ lines/cm})(3..0 \text{ cm}) = \boxed{15{,}000}$;

$R_2 = 2(5000 \text{ lines/cm})(3.0 \text{ cm}) = \boxed{30{,}000}$;

$R_3 = 3(5000 \text{ lines/cm})(3.0 \text{ cm}) = \boxed{45{,}000}$.

For the minimum-wavelength separation, we have

$\Delta\lambda = \lambda/R$;

$\Delta\lambda_1 = (420 \text{ nm})/15{,}000 = \boxed{0.028 \text{ nm}}$;

$\Delta\lambda_2 = (420 \text{ nm})/30{,}000 = \boxed{0.014 \text{ nm}}$;

$\Delta\lambda_3 = (420 \text{ nm})/45{,}000 = \boxed{0.0093 \text{ nm}}$.

8. We find the line spacing from the resolution:

$R = \lambda/\Delta\lambda = mN$;

$(530 \text{ nm})/\Delta\lambda = 3(11{,}000)$, which gives $\Delta\lambda = \boxed{0.016 \text{ nm}}$.

9. We find the minimum number of lines from the resolution:

$R = \lambda/\Delta\lambda = mN$;

$(618.3 \text{ nm})/(0.02 \text{ nm}) = (1)N$, which gives $N = \boxed{3.1 \times 10^4 \text{ lines}}$.

We find the angle for first order from

$d \sin\theta = m\lambda$;

$(4.00 \times 10^{-2} \text{ m})(3.1 \times 10^4 \text{ lines}) \sin\theta = (1)(618.33 \text{ nm})$, which gives $\sin\theta = 0.4792$, $\theta = 28.63°$.

The dispersion of the grating is

$\Delta\theta/\Delta\lambda = m/(d \cos\theta)$

$= 1/[(4.00 \times 10^{-2} \text{ m})/(3.1 \times 10^4 \text{ lines}) \cos 28.63°] = 8.83 \times 10^5 \text{ rad/m}$

$= \boxed{8.83 \times 10^{-4} \text{ rad/nm}}$.

10. (a) The average value of the two wavelengths is

The resolving power needed is

$\lambda = (454.6 \text{ nm} + 457.9 \text{ nm})/2 = 456.3 \text{ nm}$.

The resolving power needed is

$R = \lambda/\Delta\lambda = 456.3 \text{ nm}/(457.9 \text{ nm} - 454.6 \text{ nm}) = \boxed{138}$.

(b) If the width of the grarting is L, then the total number of lines in the grating is $N = L(450/\text{mm})$. Let the corresponding resolving power be at least equal to $R = 138$:

$R = mN = 2L(450/\text{mm}) = 138$;

$L = 138/[2(450/\text{mm})] = \boxed{0.153 \text{ mm}}$.

11. Consider two adjacent waves, 1 and 2, that are reflected from two adjacent lines a distance d apart on the grating. The path length difference between the two reflected waves, 1′ and 2′, is

$\Delta l = AC - BD = d(\sin\theta_r - \sin\theta_i)$.

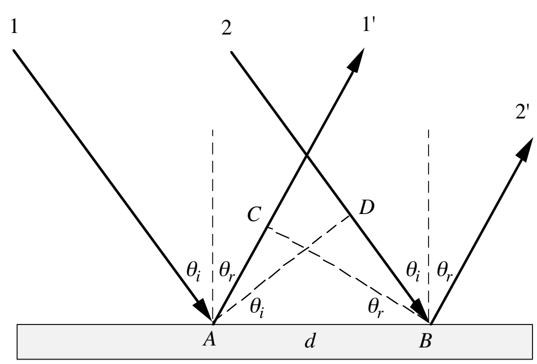
For the mth maximum, set $\Delta l = m\lambda$ to obtain

$d(\sin\theta_r - \sin\theta_i) = m\lambda$, so for $m = 2$

$\sin\theta_r = \sin\theta_i + m\lambda/d$

$= \sin 30° + 2(589 \text{ nm})/(1 \text{ cm}/4000) = 0.9712$;

$\theta_r = \boxed{76°}$.



12. We find the slit separation from

$d \sin \theta = m\lambda;$

$d \sin 0.18° = (1)(633 \text{ nm})$, which gives $d = 2.015 \times 10^5 \text{ nm} = \boxed{0.2015 \text{ mm}}$.

When the 1st and 5th slits are covered, the slit separation is not changed, so the angular position of the first order will not change: $\boxed{0.18°}$.

When the 2nd and 4th slits are covered, the slit separation will double: $d = 4.030 \times 10^5$ nm. We have

$d \sin \theta = m\lambda;$

$(4.030 \times 10^5 \text{ nm}) \sin \theta = (1)(633 \text{ nm})$, which gives $\sin \theta = 1.57 \times 10^{-3}$, $\theta = \boxed{0.090°}$.

Note that this is one-half the original angle, which we expect for small angles.

$\boxed{13}$. We find the number of lines from

$R = mN$; $10^4 = (1)N$, which gives $N = 10^4$ lines, so

$d = \ell/N = (2 \text{ cm})/(10^4) = 2 \times 10^{-4} \text{ cm} = 2 \times 10^{-6} \text{ m}.$

We find the angles for the two orders from

$\sin \theta = m\lambda/d;$

$\sin \theta_1 = (1)(580 \times 10^{-9} \text{ m})/(2 \times 10^{-6} \text{ m}) = 0.290$, which gives $\theta_1 = 16.9°$;

$\sin \theta_2 = (2)(580 \times 10^{-7} \text{ m})/(2 \times 10^{-6} \text{ m}) = 0.580$, which gives $\theta_2 = 35.5°$.

The angular separation of the two orders is $\Delta\theta = \boxed{18.6°}$.

14. We find the angle from

$\sin \theta = m\lambda/d = m\lambda/[(1 \times 10^{-2} \text{ m})/3600] = (3.60 \times 10^5 \text{ m}^{-1})m\lambda.$

The position on the screen is $y = L \tan \theta = (0.90 \text{ m}) \tan \theta$.

Blue light

$\sin \theta_1 = (3.60 \times 10^5 \text{ m}^{-1})(2)(440 \times 10^{-9} \text{ m})$, which gives $\theta_1 = 18.5°$; $y_1 = (0.90 \text{ m}) \tan \theta_1 = \boxed{0.30 \text{ m}}$.

$\sin \theta_2 = (3.60 \times 10^5 \text{ m}^{-1})(3)(440 \times 10^{-9} \text{ m})$, which gives $\theta_2 = 28.4°$; $y_2 = (0.90\text{m}) \tan \theta_2 = \boxed{0.49 \text{ m}}$.

Green light

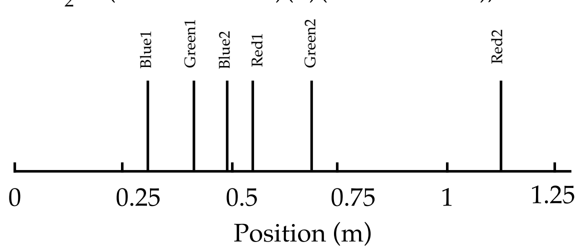
$\sin \theta_1 = (3.60 \times 10^5 \text{ m}^{-1})(2)(560 \times 10^{-9} \text{ m})$, which gives $\theta_1 = 23.8°$; $y_1 = (0.90 \text{ m}) \tan \theta_1 = \boxed{0.40 \text{ m}}$.

$\sin \theta_2 = (3.60 \times 10^5 \text{ m}^{-1})(3)(560 \times 10^{-9} \text{ m})$, which gives $\theta_2 = 37.2°$; $y_2 = (0.90 \text{ m}) \tan \theta_2 = \boxed{0.68 \text{ m}}$.

Red light

$\sin \theta_1 = (3.60 \times 10^5 \text{ m}^{-1})(2)(720 \times 10^{-9} \text{ m})$, which gives $\theta_1 = 31.2°$; $y_1 = (0.90 \text{ m}) \tan \theta_1 = \boxed{0.55 \text{ m}}$.

$\sin \theta_2 = (3.60 \times 10^5 \text{ m}^{-1})(3)(720 \times 10^{-9} \text{ m})$, which gives $\theta_2 = 51.0°$; $y_2 = (0.90 \text{ m}) \tan \theta_2 = \boxed{1.1 \text{ m}}$.



15. The slit separation is

$d = \ell/N = (1 \text{ cm})/2500 = 4 \times 10^{-4} \text{ cm} = 4 \times 10^{-6} \text{ m}.$

We find the angles from

$\sin \theta = m\lambda/d;$

$\sin \theta_{1\text{min}} = (1)(430 \times 10^{-9} \text{ m})/(4 \times 10^{-6} \text{ m}) = 0.108$, which gives $\theta_{1\text{min}} = 6.17°$;

$\sin \theta_{1\text{max}} = (1)(680 \times 10^{-7} \text{ m})/(4 \times 10^{-6} \text{ m}) = 0.170$, which gives $\theta_{1\text{max}} = 9.79°$.

The first order is $\boxed{\text{from } 6.17° \text{ to } 9.79°}$.

$\sin \theta_{2\text{min}} = (2)(430 \times 10^{-9} \text{ m})/(4 \times 10^{-6} \text{ m}) = 0.216$, which gives $\theta_{2\text{min}} = 12.42°$;

$\sin \theta_{2\text{max}} = (1)(680 \times 10^{-7} \text{ m})/(4 \times 10^{-6} \text{ m}) = 0.340$, which gives $\theta_{2\text{max}} = 19.88°$.

The second order is $\boxed{\text{from } 12.42° \text{ to } 19.88°}$.

16. We find the angles for the second-order maxima from

$d \sin \theta = m\lambda$;

$[(1\text{ cm})(10^{-2}\text{ m/cm})/(5000\text{ lines})] \sin \theta_{\text{blue}} = (2)(475 \times 10^{-9}\text{ m})$, which gives $\theta_{\text{blue}} = 28.4°$;

$[(1\text{ cm})(10^{-2}\text{ m/cm})/(5000\text{ lines})] \sin \theta_{\text{red}} = (2)(615 \times 10^{-9}\text{ m})$, which gives $\theta_{\text{red}} = 38.0°$.

The positions on the screen are

$y = L \tan \theta$;

$y_{\text{blue}} = (2.00\text{ m}) \tan 28.4° = \boxed{1.08\text{ m}}$;

$y_{\text{red}} = (2.00\text{ m}) \tan 38.0° = \boxed{1.56\text{ m}}$.

The angular width corresponds to a spread in the wavelength. We use the expression for the resolving power to find the angular width:

$R = \lambda/\Delta\lambda = mN$, or $\Delta\lambda = \lambda/mN$.

If we differentiate the condition for a maximum, $d \sin \theta = m\lambda$, we have

$d \cos \theta\, \Delta\theta = m\, \Delta\lambda = m(\lambda/mN) = \lambda/N$, which gives

$\Delta\theta = \lambda/(Nd \cos \theta) = \lambda/(W \cos \theta)$, where W is the width of the grating.

We find the width on the screen in terms of the angular width of the lines by differentiating

$y = L \tan \theta$:

$\Delta y = (L/\cos^2 \theta)\, \Delta\theta$.

The width on the screen is

$\Delta y = (L/\cos^2 \theta)\, \Delta\theta = (L/\cos^2 \theta)[\lambda/(W \cos \theta)] = L\lambda/(W \cos^3 \theta)$;

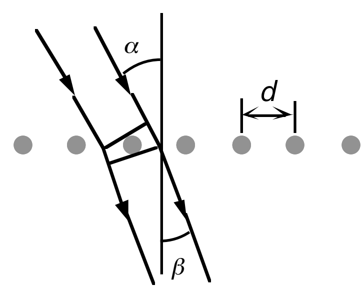
$\Delta y_{\text{blue}} = L\lambda_{\text{blue}}/(W \cos^3 \theta_{\text{blue}})$

$= (2.00\text{ m})(475 \times 10^{-9}\text{ m})/(1.2\text{ cm})(10^{-2}\text{ m/cm}) \cos^3 28.4° = 1.2 \times 10^{-4}\text{ m} = \boxed{0.12\text{ mm}}$.

$\Delta y_{\text{red}} = L\lambda_{\text{red}}/(W \cos^3 \theta_{\text{red}})$

$= (2.00\text{ m})(615 \times 10^{-9}\text{ m})/(1.2\text{ cm})(10^{-2}\text{ m/cm}) \cos^3 38.0° = 2.1 \times 10^{-4}\text{ m} = \boxed{0.21\text{ mm}}$.

17. The phase difference for rays through adjacent slits is due to the path-length difference. For a maximum, the path-length difference is a multiple of the wavelength. From the diagram, the additional distance to the grating is $d \sin \alpha$ for the ray on the right and $d \sin \beta$ for the ray on the left. For the maxima, we have



$d \sin \alpha - d \sin \beta = m\lambda,\quad m = 0, \pm 1, \pm 2, \ldots$, which gives

$\boxed{\sin \beta = \sin \alpha - m\lambda/d, \quad m = 0, \pm 1, \pm 2, \ldots}$.

18. We find the location of the dark fringes from

$a \sin \theta = m\lambda,\quad m = 0, \pm 1, \pm 2, \ldots$.

For the second dark fringe, we have

$(0.35 \times 10^{-3}\text{ m}) \sin \theta = (2)(535.0 \times 10^{-9}\text{ m})$, which gives

$\sin \theta = 3.1 \times 10^{-3}$, $\theta = \boxed{0.18°}$.

19. We find the location of the dark fringes from

$a \sin \theta = m\lambda,\quad m = 0, \pm 1, \pm 2, \ldots$;

$(2.8 \times 10^{-5}\text{ m}) \sin \theta = (1)(495 \times 10^{-9}\text{ m})$, which gives $\sin \theta = 0.018$.

The position on the screen is

$y = L \tan \theta$.

For small angles, we have

$y \simeq L \sin \theta = Lm\lambda/a$;

$\frac{1}{2}(1.8 \times 10^{-2}\text{ m}) = L(1)(495 \times 10^{-9}\text{ m})/(2.8 \times 10^{-5}\text{ m})$, which gives $L = \boxed{0.51\text{ m}}$.

20. We find the angle of the first-order maximum from

$\tan \alpha = y/L = \frac{1}{2}(6.0 \times 10^{-3}\text{ m})/(2.5\text{ m}) = 1.2 \times 10^{-3}$.

This corresponds to a small angle, so we have

$\alpha = \tan \alpha = \sin \alpha$.

If we take the maxima to be halfway between the minima, we have

$a \sin \alpha \simeq (n + \frac{1}{2})\lambda,\ \ n = \pm 1, \pm 2, \pm 3, \ldots$;

$a(1.2 \times 10^{-3}) = (1 + \frac{1}{2})(635 \times 10^{-9}\text{ m})$, which gives $a = 7.9 \times 10^{-4}\text{ m} = \boxed{0.79\text{ mm}}$.

$\boxed{21}$. (a) The minima of the diffraction pattern are found from

$a \sin \theta = m\lambda$;

$\sin \theta = m\lambda/a = m(2.6\text{ cm})/(4.2\text{ cm}) = 0.619m$. Since $|\sin \theta|$ cannot exceed 1, the only non-zero integer values of m that satisfy this equation are $m = +1$ or -1. The corresponding angles at which minima occur are

$\theta = \sin^{-1}(0.619) = \boxed{38^\circ}$ (for $m = 1$) and $\boxed{-38^\circ}$ (for $m = -1$).

As for the maxima, besides the one at $\theta = 0$, their locations are approximately determined by

$a \sin \theta = (m + \frac{1}{2})\lambda$. Thus

$\sin \theta = (m + \frac{1}{2})\lambda/a = (m + \frac{1}{2})(2.6\text{ cm})/(4.2\text{ cm}) = 0.619(m + \frac{1}{2})$.

For $m = 1$, we get $\theta = \boxed{68^\circ}$. By symmetry, another solution is $\boxed{-68^\circ}$. No other solutions are possible since they would again make $|\sin \theta|$ exceed 1.

To summarize: maxima occur at $\theta = -68^\circ, 0^\circ, 68^\circ$; and minima occur at $\theta = -38^\circ, 38^\circ$.

(b) For the secondary maximum $\theta = 68^\circ$, so its linear distance from the central maximum is

$\Delta y = (0.8\text{ m}) \tan 68^\circ = \boxed{2.0\text{ m}}$.

22. (a) We take the distance between the first minima on either side of the central maximum as the width of the central maximum. We find the position of the minimum from

$a \sin \alpha = \lambda$;

$(75 \times 10^{-6}\text{ m}) \sin \theta = 610 \times 10^{-9}\text{ m}$, which gives $\sin \theta = 0.0081$;

$y = R \tan \theta \simeq R \sin \theta = (1.20\text{ m})(0.0081) = 0.0098\text{ m}$.

The width of the central maximum is

$\Delta y = 2y = 2(0.0098\text{ m}) = 0.020\text{ m} = \boxed{2.0\text{ cm}}$.

(b) If we take the maxima to be halfway between the minima, we have

$\alpha_{max} \simeq (n + \frac{1}{2})\pi$, with $\alpha_1 = \frac{3}{2}\pi$ for the peak next to the central peak.

We find the ratio of intensities from

$I_1/I_0 = (\sin^2 \alpha_1)/\alpha_1^2 = (-1)^2/(\frac{3}{2}\pi)^2$, which gives $I_0/I_1 = \boxed{22.2}$.

23. The intensity pattern for the single slit is

$I/I_0 = (\sin^2 \alpha)/\alpha^2$,

where $\alpha = (\pi a \sin \theta)/\lambda$, and the minima occur when $\alpha_{min} = n\pi,\ \ n = \pm 1, \pm 2, \ldots$.

If we take the maxima to be halfway between the minima, we have

$\alpha_{max} \simeq (n + \frac{1}{2})\pi$, with $\alpha_1 = \frac{3}{2}\pi$ for the peak next to the central peak.

We find the ratio of intensities from

$I_1/I_0 = (\sin^2 \alpha_1)/\alpha_1^2 = (-1)^2/(\frac{3}{2}\pi)^2$, which gives $I_0/I_1 = \boxed{22.2}$.

We find the angle where the intensity is half the maximum by finding α from

$I/I_{max} = \frac{1}{2} = (\sin^2 \alpha)/\alpha^2$, or $\alpha = \sqrt{2} \sin \alpha$.

We can find α by plotting α (in rad) and $\sqrt{2} \sin \alpha$ as a function of α and finding the intersection, or by trial and error. The result is $\alpha = 1.39\text{ rad} = 79.7^\circ$.

We find the angle from

$\alpha = (\pi a \sin \theta)/\lambda$;

$1.39\text{ rad} = [\pi(10 \times 10^{-6}\text{ m}) \sin \theta]/(470 \times 10^{-9}\text{ m})$, which gives $\sin \theta = 0.0208$. $\theta = \boxed{1.19^\circ}$.

Because θ is proportional to λ and $\sin \theta$ increases as θ increases, an increase in λ will $\boxed{\text{increase } \theta}$.

24. (a) The position of the minima for a single slit are given by

$a \sin \theta = m\lambda$, or, for small angles, $\theta = m\lambda/a$, $m = \pm 1, \pm 2, \dots$.

The distance of the first minimum is

$y_1 = L \tan \theta_1 \simeq L\,\theta_1 = L\,\lambda/a$.

If we double the slit width, $a' = 2a$, we have $\boxed{y_1' = \tfrac{1}{2}y_1}$.

(b) At the central maximum all segments of the slit are in phase. The electric field amplitude will be proportional to the slit width. Because the intensity depends on (amplitude)^2, we have

$I_0'/I_0 = (E'/E)^2 = (a'/a)^2$.

If we double the slit width, $a' = 2a$, we have $\boxed{I_0' = 4I_0}$.

(c) The intensity of the first maximum will be spread over the width of the first peak on the screen. This will be true for all peaks. Because the average intensity is proportional to the maximum intensity, we have

$P'/P = (I_0'/I_0)(y_1'/y_1) = (4)(\tfrac{1}{2})$, or $\boxed{P' = 2P}$.

This is expected from doubling the area of the slit.

25. For a distant screen, the angles will be small, with $\sin\theta \simeq \tan\theta \simeq \theta$.

The position of the minima for a single slit are given by

$a \sin \theta = m\lambda$, or $\sin \theta = m\lambda/a$;

$y_m = L \tan \theta \simeq m\lambda L/a$, $m = \pm 1, \pm 2, \dots$.

The separation distance between neighboring minima is

$y_{m+1} - y_m = (m+1)\lambda L/a - m\lambda L/a = \lambda L/a$.

The separation distance between the two minima on either side of the central maximum is

$\Delta y = 2y_1 = 2(1)\lambda L/a = 2\lambda L/a$,

which is twice the separation distance between the neighboring minima.

For the double slit, the maxima are given by

$\sin \theta = m\lambda/d$, $m = 0, \pm 1, \pm 2, \dots$.

We find the minima from

$\sin \theta = (n - \tfrac{1}{2})\lambda/d$;

$y_n = L \tan \theta \simeq (n - \tfrac{1}{2})\lambda L/d$, $n = \pm 1, \pm 2, \dots$.

The separation distance between neighboring minima is

$y_{n+1} - y_n = (n + 1 - \tfrac{1}{2})\lambda L/d - (n - \tfrac{1}{2})\lambda L/d = \lambda L/d$.

The separation distance between the two minima on either side of the central maximum is

$\Delta y = 2y_1 = 2(1 - \tfrac{1}{2})\lambda L/d = \lambda L/d$,

which is the separation distance between neighboring minima.

26. For the maxima of the first wavelength, not including the central maximum, we have

$a \sin \theta = (n + \tfrac{1}{2})\lambda_1$, $n = \pm 1, \pm 2, \dots$.

For the minima of the second wavelength, we have

$a \sin \theta = m\lambda_2$, $m = \pm 1, \pm 2, \dots$.

When the second maximum, counting the central maximum as the first, coincides with the first minimum, we have

$(1 + \tfrac{1}{2})\lambda_1 = (1)\lambda_2$;

$\lambda_2 = \tfrac{3}{2}(540\text{ nm}) = \boxed{810\text{ nm}}$.

27. The angular position of the minima for a single slit are given by

$a \sin \theta = m\lambda$, or, for small angles, $\theta = m\lambda/a$, $m = \pm 1, \pm 2, \ldots$.

The angular spread of the first minimum is

$\Delta\theta_1 = 2\theta_1 = 2\lambda/a$.

To find the angular width at half-maximum, we find the phase at half-maximum:

$I = I_{\max}(\sin^2 \alpha_h)/\alpha_h^2 = \frac{1}{2}I_{\max}$, or $\alpha_h^2 = 2\sin^2 \alpha_h$.

This equation can be solved graphically or numerically to get

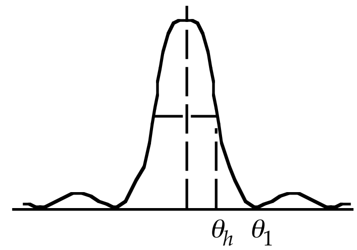
$\alpha_h = 1.392$ rad.

We find the corresponding angle from

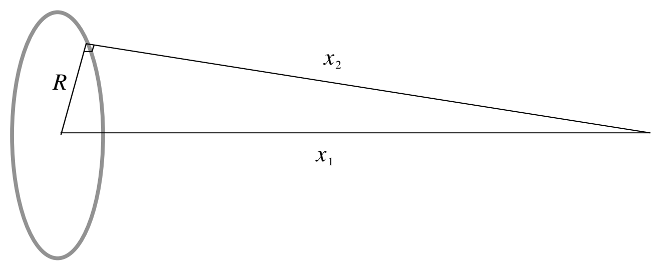
$\alpha_h = (\pi a/\lambda)\sin\theta_h \simeq \pi a\,\theta_h/\lambda$, or

$\Delta\theta_h = 2\theta_h = 2\lambda\alpha_h/\pi a = 0.886\lambda/a$.

Thus we have $\Delta\theta_h/\Delta\theta_1 = \boxed{0.443}$.



28. The diffraction minima occur where the waves from the center and the edge of the circular opening are in phase (so the sum of all the waves in between is actually zero, much like the single-slit case). So we require



$\Delta x = x_2 - x_1 = m\lambda;$

$\Delta x = (R^2 + x_1^2)^{1/2} - x_1$

$= x_1(1 + R^2/x_1^2)^{1/2} - x_1$

$\approx x_1(1 + \frac{1}{2}R^2/x_1^2) - x_1$

$= R^2/2x_1$

$= m\lambda.$

Thus the distance x_1 between the two screens at which a diffraction minimum occurs is given by

$x_1 = R^2/2m\lambda$.

Note that x_1 increases as m decreases. So the largest value of x_1 must correspond to $m = 1$. From the problem statement we know that there are two values of x_1 for which a minimum occurs. These correspond to $m = 1$ (for the second, which is also the furthest, minimum) and $m = 2$ (for the first minimum, with $x_1 = 0.60$ m). For $m = 2$ we have

$\lambda = R^2/2mx_1 = (1.1 \times 10^{-3}\text{ m})^2/[2(2)(0.60\text{ m})] = 5.0 \times 10^{-7}\text{ m} = \boxed{0.50\ \mu\text{m}}$;

and for $m = 1$

$x_1 = R^2/2m\lambda = (1.1 \times 10^{-3}\text{ m})^2/[2(1)(5.04 \times 10^{-7}\text{ m})] = \boxed{1.2\text{ m}}$.

29. The path-length difference between the top and bottom of the slit for the incident wave is

$a \sin \theta_i$.

The path-length difference between the top and bottom of the slit for the diffracted wave is

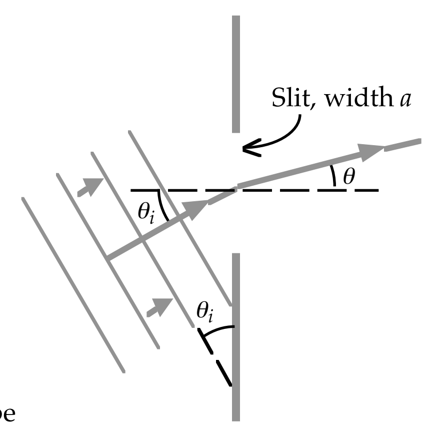
$a \sin \theta$.

When the net path-length difference is a multiple of a wavelength, there will be an even number of segments of the wave which will have a path-length difference of $\lambda/2$; there will be minima given by

$(a \sin \theta_i) - (a \sin \theta) = m\lambda$, $m = \pm 1, \pm 2, \ldots$, or

$\boxed{\sin\theta = \sin\theta_i - m\lambda/a,\ \text{where } m = \pm 1, \pm 2, \ldots}$.

When $\theta = \theta_i$, the net path-length difference is zero, and there will be constructive interference. $\boxed{\text{There is a “central maximum”}}$.



30. The total path-length difference between the rays that pass through the top and bottom of the slit is

$$(a \sin \theta) + (a \sin \phi) = a(\sin \theta + \sin \phi).$$

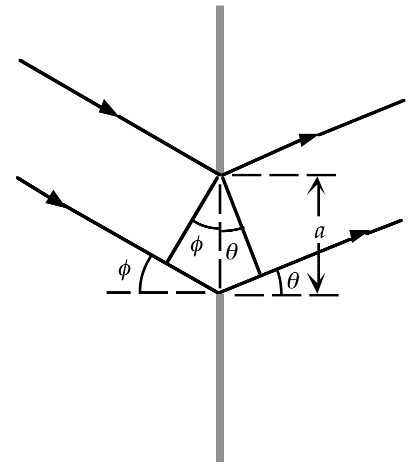
If we let 2β be the corresponding phase difference, we have

$$2\beta = (2\pi a/\lambda)(\sin \theta + \sin \phi).$$

This replaces Eq. (38–4b). In the section "How to Get the Single-Slit Intensity Pattern," with

$$\alpha = N\beta = (N\pi a/\lambda)(\sin \theta + \sin \phi),$$

the development would lead to the same Eq. (38–10).



31. (a) The intensity pattern for a single slit is

$$I = I_{max}(\sin^2 \alpha)/\alpha^2.$$

We find the location of the maxima from $dI/d\alpha = 0$:

$$dI/d\alpha = I_{max}\{[(2 \sin \alpha \cos \alpha)/\alpha^2] - [(2 \sin^2 \alpha)/\alpha^3]\} = 0, \text{ which gives}$$

$$\cos \alpha = \sin \alpha/\alpha, \quad \text{or} \quad \alpha = \tan \alpha.$$

(b) The minima of the pattern are given by

$$\sin \theta = m\lambda/a, \quad m = \pm 1, \pm 2, \ldots;$$

We find the values of α corresponding to the minima from

$$\alpha_{min} = (\pi a \sin \theta)/\lambda = m\pi, \quad m = \pm 1, \pm 2, \ldots .$$

We expect the values of α corresponding to the maxima to be approximately midway between the minima: $\alpha_{max\ 1} \simeq \frac{1}{2}(\pi + 2\pi) = \frac{3}{2}\pi$, and $\alpha_{max\ 2} \simeq \frac{1}{2}(2\pi + 3\pi) = \frac{5}{2}\pi$.

A numerical solution of $\alpha = \tan \alpha$ (remember that α is in rad) gives

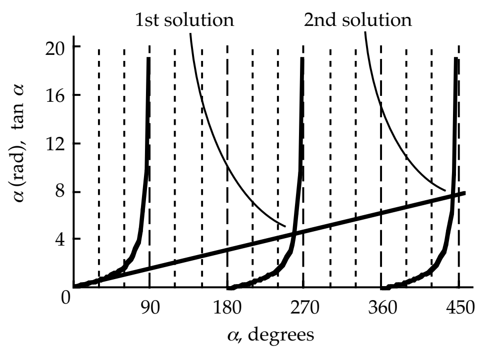
$\alpha_{max1} = 4.493$ rad = $\boxed{257.43°\text{, while midway between 1st and 2nd minima} = \tfrac{3}{2}\pi = 270°}$;

$\alpha_{max2} = 7.725$ rad = $\boxed{442.61°\text{, while midway between 2nd and 3rd minima} = \tfrac{5}{2}\pi = 450°}$.

(c) From the plot, we see that the straight line for α rises, so the intersections get closer to the angles where

$$\tan \alpha \to \infty, \alpha \to \tfrac{1}{2}\pi, \tfrac{3}{2}\pi, \tfrac{5}{2}\pi, \ldots,$$

which are the angles midway between the minima.



32. We find the angular position of the first minimum from

$$\theta_{min} \simeq 1.22\lambda/D = 1.22(2.3 \text{ cm})/(4.7 \text{ cm}) = 0.60 \text{ rad} = \boxed{34°}.$$

33. We find the minimum diameter of the telescope to resolve the two rovers from

$$\theta_{min} \simeq \lambda/D = d/R;$$

$$(650 \times 10^{-9} \text{ m})/D = (5.00 \times 10^3 \text{ m})/(3.83 \times 10^8 \text{ m}), \text{ which gives } D = 0.050 \text{ m} = \boxed{5.0 \text{ cm}}.$$

We find the minimum diameter of the telescope to detect a rover from

$$\theta_{min} \simeq \lambda/D = d/R;$$

$$(650 \times 10^{-9} \text{ m})/D = (3.0 \text{ m})/(3.83 \times 10^8 \text{ m}), \text{ which gives } D = \boxed{83 \text{ m}}.$$

34. (a) The minimum angular resolution is

$$\theta_{\min} \simeq \lambda/D = (600 \times 10^{-9}\text{ m})/(20 \times 10^{-2}\text{ m}) = \boxed{3.0 \times 10^{-6}\text{ rad} = (1.7 \times 10^{-4})°}.$$

(b) Because the images will be in the focal plane and the angle is measured from the axis, we have

$$d_{\min} = 2f\theta_{\min} = 2(2.00\text{ m})(3.00 \times 10^{-6}\text{ rad}) = 12 \times 10^{-6}\text{ m} = \boxed{12\ \mu\text{m}}.$$

(c) The minimum separation distance that can be resolved on the moon is

$$S = R\theta_{\min} = (3.83 \times 10^{8}\text{ m})(3.00 \times 10^{-6}\text{ rad}) = \boxed{1.15 \times 10^{3}\text{ m}}.$$

$\boxed{35}$. The minimum angular resolution is

$$\theta_{\min} \simeq \lambda/D = (480 \times 10^{-9}\text{ m})/(1.5 \times 10^{-3}\text{ m}) = 3.2 \times 10^{-4}\text{ rad}.$$

The separation distance is

$$S = L\theta_{\min} = (220 \times 10^{3}\text{ m})(3.2 \times 10^{-4}\text{ rad}) = \boxed{70\text{ m}}.$$

36. We estimate the separation of the stars from

$$S \simeq L\theta_{\min} = (125\text{ ly})(9.46 \times 10^{15}\text{ m/ly})(18'')(1°/3600'')[(\pi\text{ rad})/180°] = 1.0 \times 10^{14}\text{ m} = \boxed{1.0 \times 10^{11}\text{ km}}.$$

$\boxed{37}$. We find the minimum size of the aperture from

$$\theta_{\min} \simeq \lambda/D = S/L;$$

$(525 \times 10^{-9}\text{ m})/D = (10\text{ in})(2.54 \times 10^{-2}\text{ m/in})/(220\text{ mi})(1.6 \times 10^{3}\text{ m/mi})$, which gives $D = \boxed{0.73\text{ m}}$.

It would be better if the film were sensitive to $\boxed{\text{shorter}}$ wavelengths, allowing S to be smaller.

38. We find the minimum separation for resolution from

$$\theta_{\min} \simeq \lambda/D = S/L;$$

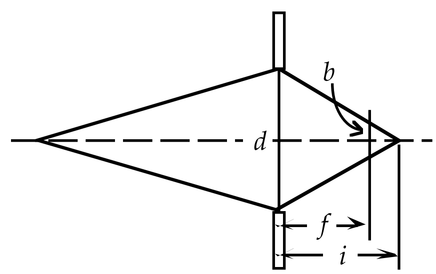
$(550 \times 10^{-9}\text{ m})/(5.0 \times 10^{-3}\text{ m}) = S/(1.0 \times 10^{3}\text{ m})$, which gives $D = 0.11\text{ m} = \boxed{11\text{ cm}}$.

39. The image distance for a distant object will be the focal length.
We find the image distance for the object 5.0 m away from

$(1/s) + (1/i) = 1/f$:

$(1/5.0\text{ m}) + (1/i) = 1/0.050\text{ m}$, which gives

$i = 0.0505\text{ m} = 50.5\text{ mm}$.



If the aperture is d, from similar triangles we find the diameter of the image on the film b from

$d/i = b/(i - f)$, which gives $b = d(i - f)/i$.

We estimate the diameter of the diffraction circle b' from

$\theta_{\min} \simeq \lambda/d = \frac{1}{2}b'/f$, which gives $b' = 2\lambda f/d$.

For the two sources to have equal contributions, we use an average wavelength of 550 nm and have

$b = b'$; $d(i - f)/i = 2\lambda f/d$, or

$d^2 = 2\lambda fi/(i - f) = 2(550 \times 10^{-9}\text{ m})(50 \times 10^{-3}\text{ m})(50.5\text{ mm})/(50.5\text{ mm} - 50\text{ mm})$, which gives

$d = 2.4 \times 10^{-3}\text{ m} = \boxed{2.4\text{ mm}}$.

40. We use 2 cm for the headline and 2 mm for the diameter of the pupil. We estimate the distance from

$$\theta_{\min} \simeq \lambda/D = S/L;$$

$(600 \times 10^{-9}\text{ m})/(2 \times 10^{-3}\text{ m}) = (2 \times 10^{-2}\text{ m})/L$, which gives $L \simeq \boxed{60\text{ m}}$.

41. We find the minimum separation distance that can be resolved from

$$\theta_{\min} \simeq \lambda/D = S/L;$$

$(520 \times 10^{-9}\text{ m})/(1.5 \times 10^{-3}\text{ m}) = S/(60{,}000\text{ ft})$, which gives $S = \boxed{21\text{ ft}}$.

42. We find the distance of approach from

$\theta_{\min} \simeq \lambda/D = S/L$;

$(510 \times 10^{-9}\text{ m})/(4.2 \times 10^{-3}\text{ m}) = (1.8\text{ m})/L$, which gives $L = 1.5 \times 10^4\text{ m} = \boxed{15\text{ km}}$.

43. The maxima of the grating are given by

$\beta = (\pi d \sin\theta)/\lambda = n\pi,\ \ n = 0, \pm 1, \pm 2, \dots$.

The minima of the single-slit pattern are given by

$\alpha = (\pi a \sin\theta)/\lambda = m\pi,\ \ m = \pm 1, \pm 2, \dots$.

The angles where the maxima of the grating fall on the minima of the single slit are given by

$\sin\theta = n\lambda/d = m\lambda/a$, which becomes

$n/m = d/a = 3$.

For the first minimum of the single slit, $m = 1$, we have $n = 3$, so the $\boxed{\text{third order}}$ is missing.

44. The location of the tenth maximum is given by

$d \sin\theta = m\lambda = 9\lambda$.

The phase at the tenth maximum of the double-slit pattern is

$\beta = (\pi d \sin\theta)/\lambda = 9\pi$,

and the phase there for the single-slit pattern is

$\alpha = (\pi a \sin\theta)/\lambda = (\pi a/\lambda)(9\lambda/d) = (9a/d)\pi = 0.9\pi = 162°$.

At any maximum of the double-slit pattern, we have

$\sin(2\beta)/\sin\beta = (2\sin\beta\cos\beta)/\sin\beta = 2\cos m\pi = \pm 2$,

so the variation of intensity at the maxima is due to the single-slit variation only.

At the central maximum, we have

$(\sin 0°)/0° = 1$,

so the ratio of the intensities is given by

$I/I_{\max} = [(\sin\alpha)/\alpha]^2/[(\sin 0°)/0°] = [\sin(162°)/0.9\pi]^2/1$, which gives $I/I_{\max} = \boxed{0.012}$.

$\boxed{45}$. Because the angles are small, we have

$\sin\theta \simeq \theta \simeq \tan\theta = y/R$

$= y/(3.0 \times 10^3\text{ mm}) = 3.33 \times 10^{-4}\,y$, with y in mm.

The phase for the double-slit pattern is

$\beta = (\pi d \sin\theta)/\lambda \simeq \pi d\,\theta/\lambda$

$= \pi(1.1 \times 10^{-3}\text{ m})(3.33 \times 10^{-4}\,y)/(690 \times 10^{-9}\text{ m}) = 1.67\,y$, with y in mm;

and the phase for the single-slit pattern is

$\alpha = (\pi a \sin\theta)/\lambda \simeq \pi a\,\theta/\lambda$

$= \pi(0.20 \times 10^{-3}\text{ m})(3.33 \times 10^{-4}\,y)/(690 \times 10^{-9}\text{ m}) = 0.303\,y$, with y in mm.

The intensity of the pattern is

$I = I_0[\sin(2\beta)/\sin\beta]^2\,[(\sin\alpha)/\alpha)]^2 = 4I_0(\cos^2\beta)[(\sin\alpha)/\alpha]^2$.

For the four locations, we have

y, mm	θ, rad	α, rad	β, rad	I/I_0
0.050	1.67×10^{-5}	0.0152	0.0835	$\boxed{3.97}$
0.50	1.67×10^{-4}	0.152	0.835	$\boxed{1.79}$
1.5	5.00×10^{-4}	0.455	2.51	$\boxed{2.43}$
3.0	1.00×10^{-3}	0.909	5.01	$\boxed{0.26}$

46. For the small angles shown, the maxima of the grating are given by

$\sin\theta \simeq \theta \simeq n\lambda/d,\ \ n = 0, \pm 1, \pm 2, \dots$.

We use the data from the fifth peak:

$0.01\text{ rad} = (5)(550\text{ nm})/d$, which gives $d \simeq 3 \times 10^5\text{ nm} = \boxed{0.3\text{ mm}}$.

We see that the third and sixth orders are missing. From Problem 43 we have

$d/a = n/m = 3/1 = 6/2 = 3$, so $a = d/3 = \boxed{0.1\text{ mm}}$.

47. The multiple slit pattern has two small peaks between the large peaks. The smaller peaks occur when there is constructive interference between non-adjacent slits, but not between adjacent slits, so the intensity is much less. Because there are two peaks, there must be three spacings or four slits. This can be seen from the plot of $I = I_{max}[\sin(N\beta)/\sin\beta]^2$ for $N = 4$ shown in Figure 39–9.
The larger peaks occur when the rays from all slits are in phase, so the path difference between adjacent slits is $n\lambda$. This is the same as the condition for the double slit. Because these peaks are at the same angles as those in Problem 40, the slit width and separation are the same:
$a = \boxed{0.1\text{ mm}}$ and $d = \boxed{0.3\text{ mm}}$.

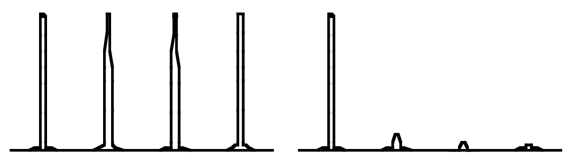
48. (a) The maxima of the grating are given by
$\sin\theta = n\lambda/d,\ n = 0, \pm 1, \pm 2, \ldots$.
For adjacent maxima, $\Delta n = 1$, so we have
$\lambda/d = \Delta(\sin\theta)$;
$(600 \times 10^{-9}\text{ m})/d = 0.36 - 0.30$, which gives $d = 1.0 \times 10^{-5}\text{ m} = \boxed{10\ \mu\text{m}}$.
(b) The minima of the single-slit pattern are given by
$\sin\theta = m\lambda/a,\ m = \pm 1, \pm 2, \ldots$.
Because the fourth order of the double slit is missing, we have
$\sin\theta_5 = 4\lambda/d = m\lambda/a$.
The smallest slit width will have the smallest value of $m = 1$;
$a = md/4 = (1)(10\ \mu\text{m})/4 = \boxed{2.5\ \mu\text{m}}$.
(c) Because all orders which are multiples of 5 will be missing, we have
$m = \boxed{1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15, 16, \ldots}$.

49. The maxima of the grating are given by
$\sin\theta = n\lambda/d,\ n = 0, \pm 1, \pm 2, \ldots$.
The minima of the single-slit pattern are given by
$\sin\theta = m\lambda/a,\ m = \pm 1, \pm 2, \ldots$.
Orders will be missing if
$n\lambda/d = m\lambda/a$, or $n = (d/a)m$.
Because $d/a = (1.2\text{ mm})/(0.4\text{ mm}) = 3$ is an integer, the missing orders are
$n = 3m = \boxed{3, 6, 9, \ldots}$.
Because the angles are small, we find the angles for the missing orders from
$\sin\theta \simeq \theta = m\lambda/a = m(589 \times 10^{-9}\text{ m})/(0.4 \times 10^{-3}\text{ m}) = (1.47 \times 10^{-3})m$ rad, $m = 3, 6, 9, \ldots$.
The angles are $\boxed{0.084°, 0.169°, 0.253°, \ldots}$.

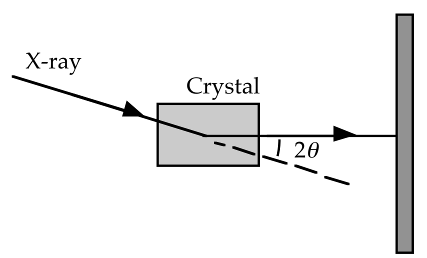
50. The locations of the maxima of the grating are given by
$d\sin\theta = m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$.
At the maxima, the phase for the diffraction grating pattern is $\beta = (\pi d\sin\theta)/\lambda = m\pi$,
and the phase for the single-slit pattern is $\alpha = (\pi a\sin\theta)/\lambda = \pi(a/d)m = \pi(1/3)m$.
At a maximum of the diffraction grating pattern, we have
$\sin(N\beta)/\sin\beta = N$.
The intensity distribution on the screen is given by
$I = I_{max}[\sin(N\beta)/\sin\beta]^2\,[(\sin\alpha)/\alpha]^2$,
so the ratio of the second-order and first-order maxima is
$$I_2/I_1 = I_{max}(\sin N\beta_2/\sin\beta_2)^2\,(\sin\alpha_2/\alpha_2)^2/I_{max}(\sin N\beta_1/\sin\beta_1)^2\,(\sin\alpha_1/\alpha_1)^2$$
$$= \{[\sin(2\pi/3)]/(2\pi/3)\}^2/\{[\sin(\pi/3)]/(\pi/3)\}^2$$
$$= \boxed{1/4}.$$

51. The phase for the double-slit pattern is
$\beta = (\pi d \sin\theta)/\lambda \simeq \pi d\,\theta/\lambda,$
and the phase for the single-slit pattern is
$\alpha = (\pi a \sin\theta)/\lambda \simeq \pi a\,\theta/\lambda$, which gives
$\beta/\alpha = d/a = (0.30 \text{ mm})/(0.25 \text{ mm}) = 1.20.$
The intensity of the pattern is
$I = I_0[\sin(2\beta)/(\sin\beta)]^2\,[(\sin\alpha)/\alpha)]^2 = 4I_0(\cos^2\beta)[(\sin\alpha)/\alpha]^2.$
We find the value of α from
$I = \frac{1}{2}I_{\max} = 2I_0$, or $(\cos^2\beta)[(\sin\alpha)/\alpha]^2 = 1/2.$
If we take the square root and use $\beta = 1.20\alpha$, we get a transcendental equation for α:
$\alpha/\sqrt{2} = \cos(1.20\alpha)\sin\alpha.$
A numerical solution gives $\alpha = 0.600$ rad.
We find the angle from the central axis from
$\sin\theta = \lambda\alpha/\pi a = (625 \times 10^{-9} \text{ m})(0.600 \text{ rad})/\pi(0.25 \times 10^{-3} \text{ m}) = 0.000478,$
which gives $\theta = 4.78 \times 10^{-4}$ rad $= \boxed{0.0274°}$.

52. The phase for the double-slit pattern is
$\beta = (\pi d \sin\theta)/\lambda,$
and the phase for the single-slit pattern is
$\alpha = (\pi a \sin\theta)/\lambda$, which gives
$\beta/\alpha = d/a.$
(a) If $d >> a$, we have $\beta >> \alpha$, which means that many maxima of the double-slit pattern occur within the central single-slit maximum.
(b) If $d - a << a$, we have $\beta - \alpha << \alpha$, or $\beta << 2\alpha$, which means that the maxima of the double-slit pattern
occur where intensity of the central single-slit maximum is very small.


(a) (b)

53. We find the Bragg-plane spacing from
$2d\sin\theta = n\lambda;$
$2d\sin[\frac{1}{2}(38.2°)] = (1)(0.14 \text{ nm}),$
which gives $d = \boxed{0.21 \text{ nm}}$.



54. We find the angles for the diffraction peaks from
$\sin\theta_n = n\lambda/2d,\ n = 1, 2, 3, \ldots;$
$\sin\theta_n = n(0.55 \text{ nm})/[2(0.30 \text{ nm})] = 0.92n;$
$\sin\theta_1 = 0.92(1) = 0.92$, which gives $\theta_1 = \boxed{66°}$;
$\sin\theta_2 = 0.92(2) = 1.84$, so $\boxed{\theta_2 \text{ does not occur}}$.

55. We find the angle for the diffraction peak from
$\sin\theta_n = n\lambda/2d,\ n = 1, 2, 3, \ldots;$
$\sin\theta_1 = (1)(0.14 \text{ nm})/2(0.28 \text{ nm}) = 0.25$, which gives $\theta_1 = 14.5°$.
The scattering angle is $2\theta_1 = \boxed{29.0°}$.

56. The diffraction angles are found from

$\sin \theta = n\lambda/2d.$

Because $\sin \theta$ increases with θ, we see that a smaller d gives a larger angle, so $\boxed{\text{rock salt}}$ will give a greater angular separation. We find the angles for the two wavelengths scattered from mica from

$\sin \theta_1 = \lambda_1/2d = (0.096 \text{ nm})/2(1.0 \text{ nm}) = 0.048$, which gives $\theta_1 = 2.75°$;

$\sin \theta_2 = \lambda_2/2d = (0.104 \text{ nm})/2(1.0 \text{ nm}) = 0.052$, which gives $\theta_2 = 2.98°$.

The angular separation is $\Delta\theta_{mica} = \boxed{0.23°}$.

We find the angles for the two wavelengths scattered from rock salt from

$\sin \theta_1 = \lambda_1/2d = (0.096 \text{ nm})/2(0.28 \text{ nm}) = 0.171$, which gives $\theta_1 = 9.87°$;

$\sin \theta_2 = \lambda_2/2d = (0.104 \text{ nm})/2(0.28 \text{ nm}) = 0.186$, which gives $\theta_2 = 10.70°$.

The angular separation is $\Delta\theta_{\text{rock salt}} = \boxed{0.83°}$.

57. We estimate the angular spread of the beam from

$\theta_{min} \simeq \lambda/D = \lambda/(4A/\pi)^{1/2} = (690 \times 10^{-9} \text{ m})/[4(1.0 \times 10^{-3} \text{m}^2)/\pi]^{1/2} = 1.9 \times 10^{-5}$ rad.

The minimum diameter of the beam at the moon is

$S_{min} = 2L\theta_{min} = 2(3.84 \times 10^8 \text{ m})(1.9 \times 10^{-5} \text{ rad}) = \boxed{15 \times 10^3 \text{ m}}$.

58. We find the wavelength spread from the resolving power:

$R = mN = \lambda/\Delta\lambda;$

$(3)(16{,}000) = (623.000 \text{ nm})/\Delta\lambda$, which gives $\Delta\lambda = 0.013$ nm.

The wavelength of the second line is

$\lambda_2 = \lambda_1 + \Delta\lambda = \boxed{623.013 \text{ nm}}$.

59. (a) Eestimate the angular resolution from the angle subtended by a length of 1 m at the radar system:

$\theta \simeq S/L = (1 \text{ m})/(100 \times 10^3 \text{ m}) = \boxed{1.0 \times 10^{-5} \text{ rad} = (5.7 \times 10^{-4})°}$.

(b) We estimate the wavelength from

$\theta \simeq \lambda/D;$

$1.0 \times 10^{-5} \text{ rad} = \lambda/(2.5 \text{ m})$, which gives $\lambda = 2.5 \times 10^{-5} \text{ m} = \boxed{25\ \mu\text{m}.}$

60. This situation is equivalent to a single-slit diffraction. We find the first minimum from

$\sin \theta = m\lambda/d;$

$\sin 50° = (1)\lambda/(50 \text{ m})$, which gives $\lambda = \boxed{38 \text{ m}}$.

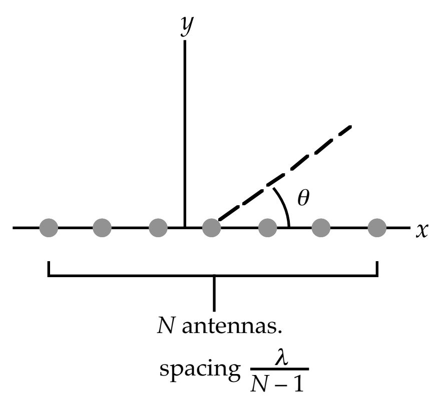
61. (a) This system is equivalent to a grating of N lines with a spacing of $d = \lambda/(N-1)$. Because the angle is measured from the normal to the grating, the phase for this array is

$\beta = [\pi d \sin(90° - \theta)]/\lambda = (\pi d \cos \theta)/\lambda = (\pi \cos \theta)/(N-1).$

Because the antennas are perfect radiators, there is no single-slit effect. The intensity is

$I = I_0[\sin(N\beta)/\sin \beta]^2$

$= \boxed{I_0\{\sin[(N\pi \cos \theta)/(N-1)]/\sin[(\pi \cos \theta)/(N-1)]\}^2}$.



(b) The maxima of the pattern occur at

$\sin(90° - \theta) = \cos \theta = m\lambda/d = m(N-1),\ m = 0, \pm 1, \pm 2, \ldots$.

Because $N > 1$, and $\cos \theta \leq 1$, the only possible value of m is 0. The maxima occur at

$\cos \theta = 0,\ \ \theta = \pi/2$ and $3\pi/2$, with a magnitude of $I = N^2 I_0$.

At $\theta = 0$ and 180°, we have

$I = I_0\{\sin [N\pi/(N-1)]/\sin [\pi/(N-1)]\}^2 \to I_0$, for $N > 2$.

$\boxed{\text{There is a slight decrease in } I \text{ for } \theta \text{ close to } 0° \text{ and } 180° \text{ and a buildup to the maxima at } 90° \text{ and } 270°.}$

62. Let the width of each clear slit be a. The diffraction minima are given by

$a \sin \theta = m\lambda$.

If the width of each opaque space is b, then the center-to-center spacing between adjacent clear slits is $d = a + b$, and the interference maxima are determined by

$d \sin \theta = (a + b) \sin \theta = m'\lambda$.

If $d = 2a$ then the diffraction minima satisfy

$d \sin \theta = 2a \sin \theta = 2m\lambda$, which happens to be the same equation that determines the location of the $2m$-th interference maximum ($m' = 2m$). Therefore, if $d = 2a$ then all the even-order interference maxima would be missing as each of them coincides with a diffraction minimum. Thus

$a/b = a/(d - a) = a/(2a - a) = \boxed{1}$.

63. If the wavelength of the light used in the experiment is λ_0 in air, then its value in the glass is $\lambda = \lambda_0/n$. The equation that determines the mth interference maximum in the glass is now

$a \sin \theta = m\lambda = m\lambda_0/n$.

As the light rays refract out of the glass onto the screen in air, however, θ changes to θ_0 according to Snell's law:

$\sin \theta = \sin \theta_0/n$.

Thus the angular location of the mth interference maximum that shows up on the screen in air satisfies

$a \sin \theta = a(\sin \theta_0/n) = m\lambda_0/n$, or

$a \sin \theta_0 = m\lambda_0$.

The last equation is identical to the case when the glass is absent. The pattern on the screen does not change at all as a result of the grating being embedded in the glass. So $\boxed{\text{no}}$, this method cannot yield a measurement of the index of refraction of the glass.

64. The relation between the energy and momentum of the electron is $E = p^2/2m$.
The associated wavelength is

$\lambda = h/p = h/(2mE)^{1/2}$.

We estimate the smallest angular separation from

$\theta_{\text{min}} \simeq \lambda/D = [h/(2mE)^{1/2}]/D = h/D(2mE)^{1/2}$

$= (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})/(0.06 \times 10^{-3}\ \text{m})[2(9.1 \times 10^{-31}\ \text{kg})(8.0 \times 10^3\ \text{eV})(1.6 \times 10^{-19}\ \text{J/eV})]^{1/2}$

$= \boxed{2.3 \times 10^{-7}\ \text{rad} = (1.3 \times 10^{-5})^\circ}$.

65. From Babinet's principle, we know that the size of the circular diffraction pattern produced by the obstacle is the same as that produced by an opening the same size as the obstacle. The angle from the axis subtended by the spot on the screen is

$\theta \simeq \tan \theta = \frac{1}{2}D/L$.

We estimate the size of the obstacle d from

$\theta \simeq \lambda/d = \frac{1}{2}D/L$;

$(633 \times 10^{-9}\ \text{nm})/d = \frac{1}{2}(0.70 \times 10^{-2}\ \text{m})/(2.5\ \text{m})$, which gives $d = 4.5 \times 10^{-4}\ \text{m} = \boxed{0.45\ \text{mm}}$.

66. From Babinet's principle, we know that the diffraction pattern is the same as that of a diffraction grating with spacing d. The maxima of the pattern will be given by

$\boxed{\sin \theta = m\lambda/d,\ m = \pm 1, \pm 2, \ldots}$.

67. The wavelength of the radiation is

$\lambda = c/f = (3 \times 10^8\ \text{m/s})/(1.25 \times 10^{23}\ \text{Hz}) = 2.4 \times 10^{-15}\ \text{m}$.

Because this is comparable to the size of the nucleus, we do not expect the first minimum to be at a small angle. From Babinet's principle, we estimate the angle from the first minimum of the diffraction pattern of a single slit the size of the nucleus:

$\sin \theta \simeq \lambda/a = (2.4 \times 10^{-15}\ \text{m})/[2(3.2 \times 10^{-15}\ \text{m})] = 0.375$, which gives $\theta = \boxed{22^\circ}$.

From the discussion of the Airy disk, the value obtained by using a factor of 1.22 in the expression for $\sin \theta$ is 27°.